Observational Constraints on the Internal Structure and Dynamics of the Vela Pulsar

Mark Abney,[1,2] Richard I. Epstein,[1] and Angela V. Olinto[1,2]

[1]Los Alamos National Laboratory, Los Alamos, NM 87545, USA

[2]Department of Astronomy and Astrophysics, and Enrico Fermi Institute,

University of Chicago, 5640 S. Ellis Ave, Chicago, IL 60637

ABSTRACT

We show that the short spin-up time observed for the Vela pulsar during the 1988 "Christmas" glitch implies that the coupling time of the pulsar core to its crust is less than ~ 10 seconds. Ekman pumping cannot explain the fast core-crust coupling and a more effective coupling is necessary. The internal magnetic field of the Vela pulsar can provide the necessary coupling if the field threads the core with a magnitude that exceeds 10^{13} Gauss for a normal interior and 10^{11} Gauss for a superconducting interior. These lower bounds favor the hypothesis that the interior of neutron stars contains superfluid neutrons and protons and challenge the notion that pulsar magnetic fields decay over million year time scales or that magnetic flux is expelled from the core as the star slows.

Subject headings: stars: evolution — stars: interiors — magnetic fields — stars: neutron

1.   The Vela Christmas Glitch

Observations of the Vela pulsar during the December 24, 1988 "Christmas" glitch provide a glimpse of the coupling between the solid crust and the fluid interior of a neutron star. The phase residuals for this event are shown in Figure 1 (McCulloch *et al.* 1990). The nearly linear decay in the residuals immediately after the glitch shows that the angular velocity of the surface first changed abruptly - within the ~ 120 s time resolution of the observations - and then varied much more slowly. As we show below, this abrupt change strongly constrains the internal structure of the Vela pulsar.

A neutron star is composed of a solid crust containing at most a few percent of the star's moment of inertia, a liquid core, and a superfluid neutron liquid that coexists with the crystal lattice of the inner part of the crust. If both the neutrons and protons in the core are superfluid, they would be strongly coupled by Fermi liquid effects (Alpar, Langer & Sauls 1984; Alpar & Sauls 1988). This quantum fluid together with the electrons, which are needed to maintain charge neutrality, act as a single classical liquid whose viscosity is supplied mainly by the electrons. In this Letter, we study the coupling of this core liquid to the star's crust.

In current models of pulsar glitches (Pines & Alpar 1985, Link & Epstein 1996), the observed spin up is due to the transfer of angular momentum from the neutron superfluid in the inner-crust to the solid crust. In these "crust-initiated" models, subsequent exchange of angular momentum between the crust and the core brings these two components into rotational equilibrium. In alternative "core-initiated" models, the initial spin-up occurs in the core rather than the crust (Sauls 1989) and the crust then catches up to the core's angular velocity.

Models in which glitches are almost entirely a crustal phenomenon, with only weak coupling to the core, are ruled out by timing data from accreting neutron stars. This type of "crust-only" mechanism would require that the crust of the Vela pulsar remains decoupled from the core for months, the characteristic post-glitch relaxation time scale. However, analyses of timing noise in accretion-powered pulsars (Boynton & Deeter 1979; Boynton 1981; Boynton *et al.* 1984) indicate

that $\gtrsim 14\%$ of the stellar core couples to the crust in much less than a month, ruling out the possibility of "crust-only" glitch mechanisms.

For either the crust-initiated or core-initiated models, the Vela Christmas glitch strongly constrains the core-crust coupling time-scale. A linear coupling model for crust-interior interaction can illustrate the types of constraints different glitch models yield. If I_C and $\Omega_C(t)$ are the moment of inertia and angular velocity of the solid crust, I_I and $\Omega_I(t)$ are the moment of inertia and angular velocity of the liquid interior (which we take to behave as a solid body for this schematic example), and t_{CI} is the coupling time scale between the crust and the interior, we can write:

$$I_C\dot{\Omega}_C(t) = -\dot{J}_{CI}(t) + \dot{J}_{C,S}(t) \ , \tag{1}$$

$$I_I\dot{\Omega}_I(t) = \dot{J}_{CI}(t) + \dot{J}_{I,S}(t) \ , \tag{2}$$

where

$$\dot{J}_{CI}(t) \equiv I_r\frac{\Omega_C(t) - \Omega_I(t)}{t_{CI}}, \tag{3}$$

and

$$I_r = \frac{I_C I_I}{(I_C + I_I)}$$

is the reduced moment of inertia. For a crust-initiated glitch the source term is $\dot{J}_{C,S}(t) = \tau_C$, for $0 < t < t_{su}^C$ (and $\dot{J}_{I,S}(t) = 0$); whereas for a core-initiated glitch $\dot{J}_{I,S}(t) = \tau_I$, for $0 < t < t_{su}^I$ (and $\dot{J}_{C,S}(t) = 0$), where the torques τ_C and τ_I are taken to be constants.

We solve the linear coupling model for both a core- and crust-initiated glitch and obtain the phase residuals:

$$\Delta\phi(t) = \int_0^t [\Omega_C(0) - \Omega_C(t')]dt'. \tag{4}$$

For $t \geq t_{su}^C$ and $t \gg t_{CI}$, the phase residuals for a crust-initiated glitch are given by,

$$\Delta\phi_C(t) = -\frac{\tau_C\, t_{su}^C}{I_C + I_I}\left(t - \frac{t_{su}^C}{2} + \frac{I_I}{I_C}t_{CI}\right) \ , \tag{5}$$

while, for a core-initiated glitch and $t \geq t_{su}^I$,

$$\Delta\phi_I(t) = -\frac{\tau_I\, t_{su}^I}{I_C + I_I}\left(t - \frac{t_{su}^I}{2} - t_{CI}\right) \ . \tag{6}$$

By fitting the data from the Christmas glitch to the above equations, taking into account the uncertainty in when the glitch started, we constrain the coupling and spin-up times. The short-term behavior of the glitch model enables us to place upper limits on these time scales by requiring that the calculated phase residuals agree with the data to within the observational uncertainty (~ 0.2 milliperiods). Acceptable models lie between the curves shown in Figure 1, where the dashed curves show the phase residuals for two crust-initiated glitch models and the solid curves show results for two core-initiated models.

We find that for a crust-initiated glitch the core-crust coupling time scale must satisfy

$$t_{CI} \lesssim 300\,\text{s}\,\frac{I_C}{I_I}\,, \tag{7}$$

and the spin-up time must be $t_{su}^C \lesssim 1200$ s [this constraint on the spin-up time is in agreement with the theoretical models of Link and Epstein (1996)]. Since $I_C/I_I \sim 0.03$, we have

$$t_{CI} \lesssim 10\,\text{s}\,. \tag{8}$$

The upper bound on the coupling time scale is much shorter than the experimental resolution of the spin-rate changes due to the small moment of inertia of the crust.

Crust-initiated models have shown excellent agreement with the observed behavior of glitching pulsars (Link & Epstein 1996) and will be the main focus of the discussion below. For completeness, we point out that for core-initiated glitch models the requirement for the core-crust coupling time scale is less severe, $t_{CI} \lesssim 440$ s, and the spin-up time must satisfy $t_{su}^I \lesssim 1200$ s.

2.   Fluid Dynamics in Neutron Star Cores

The requirement that the neutron star core and crust couple in less than 10 s sets the most stringent constraints yet on the internal dynamics of these objects. We first review the proposed mechanisms for dynamically coupling the components of a neutron star and then examine the physics needed to explain the Christmas glitch.

Easson (1979) suggested that Ekman pumping may rapidly bring crust and core into corotation. In Ekman pumping, angular momentum is transferred via an imbalance between pressure and centrifugal forces in a thin boundary layer. This imbalance drives a circulation from the crust to the interior. If this mechanism worked as Easson argued, it would couple the crust to the interior on time scale

$$t_E \simeq \frac{1}{2\Omega_C E^{1/2}} \simeq 250 \text{ seconds for Vela}, \tag{9}$$

where $E = \nu/2\Omega_C R^2$ is the Ekman number, R is the radius of the star $(R \sim 10^6 \text{cm})$, and ν is the kinematic viscosity from Easson and Pethick (1979). If Ekman pumping worked, this timescale would be too long for crust-initiated glitches but still adequate for core-initiated glitches.

Abney and Epstein (1996) reinvestigated Ekman pumping taking into account the effects of compressibility and composition stratification. They showed that in neutron stars the combined effects of stratification and compressibility restrict the Ekman pumping process to a relatively thin zone near the boundary, leaving much of the interior fluid unaffected and greatly increasing the spin-up time.

Following Abney and Epstein (1996), we define the dimensionless "constant-Y compressibility" as $\kappa_Y \equiv gR/c_Y^2$, where g is the gravitational acceleration and c_Y is the speed that relates the change in pressure, p, with the change in density, ρ, when an element of fluid is displaced while holding a parameter Y constant, i.e.;

$$\left(\frac{\partial \rho}{\partial p}\right)_Y \equiv \frac{1}{c_Y^2}. \tag{10}$$

If the fluid is displaced adiabatically so that the entropy and composition are fixed, then c_Y is the sound speed. For a typical neutron star, $g \approx 10^{14} \text{cm/sec}^2$ and $c_Y \approx 10^9 \text{cm/sec}$ (Epstein 1988), therefore, $\kappa_Y \sim 10^2$.

The effects of stratification are characterized by the Brunt–Väisälä frequency N given by

$$N^2 \equiv g^2 \left(\frac{1}{c_{\text{eq}}^2} - \frac{1}{c_Y^2}\right), \tag{11}$$

where c_{eq} which is slightly less than c_Y relates the change in density to the change in pressure

through the equilibrium stellar interior,

$$\left(\frac{\partial \rho}{\partial p}\right)_{\mathrm{eq}} \equiv \frac{1}{c_{\mathrm{eq}}^2}. \tag{12}$$

For compressibilities of $\kappa_Y \sim 10^2$, Abney & Epstein (1996) find that the Brunt–Väisälä frequency, N has to be $\lesssim 0.2\,\Omega_C$ for a significant fraction of the star to undergo Ekman pumping.

The Brunt–Väisälä frequency for the core of neutron stars has been studied by Reisenegger and Goldreich (1992) and Lee (1995). The neutron-to-proton ratio of a fully relaxed neutron star is set by an equilibrium among strong and weak interactions that minimizes the free energy. If two mass elements from different depths in the neutron star are interchanged, the composition of the mass elements would revert to those of the local ambient material after a weak interaction time scale and there would be no restoring force. However, for shorter times the material "remembers" its origin (i.e., the neutron-to-proton ratio does not fully adjust) and a restoring force pulls on the displaced matter. Since the dynamic time scale for perturbations in a neutron star are much shorter than the weak-interaction time scales, the variation of the neutron-to-proton ratio with depth stabilizes radial motions generating a finite Brunt–Väisälä frequency. Using the Pandharipande (1971) equation of state, Reisenegger and Goldreich (1992) estimated

$$N \approx 0.05 \left(\frac{\rho}{\rho_{\mathrm{nuc}}}\right)^{1/2} \frac{g}{c_{\mathrm{eq}}} \tag{13}$$

which gives $N \approx 500\,s^{-1}$ around nuclear density and $N \approx 7\,\Omega_C$ for Vela. Lee (1995) used the Serot (1979) equation of state to study the radial dependence of N, and found a similar values for N in the outer regions of the core.

Alternatively, the core of neutron stars may be composed of quark matter (Ivanenko & Kurdgelaidze 1969, Itoh 1970, Chapline & Nauenberg 1977, Freedman & McLerran 1978, Fechner & Joss 1978). In this case, we can use the strange matter equation of state (Witten 1984, Haensel, Zdunik, & Schaeffer 1986, Alcock, Farhi, & Olinto 1986) to calculate the Brunt–Väisälä frequency for stars composed mostly of quarks. We find that for a gas of massless up and down quarks and

strange quarks with mass m_s,

$$N \simeq 0.3 \frac{g}{c_{\text{eq}}} \left(\frac{m_s}{205 \text{MeV}} \right)^2 \tag{14}$$

for a bag constant $B = (145 \text{MeV})^4$. The radial dependence of N is quite mild in this case varying at most by a factor of a few over 10 km for a 1.4 $M_\odot$ quark star. Since $m_s \gg 14$ MeV, $N \gg 0.2\,\Omega_C$ for Vela and Ekman pumping is inhibited.

3. Vela's Internal Magnetic Field

In the absence of Ekman pumping, viscosity couples the core and crust on a time scale

$$t_{vis} \sim \frac{1}{\Omega_C E} \sim \text{ months.} \tag{15}$$

This time is too long to explain the observed behavior during the Christmas glitch and an alternative mechanism for the core-crust coupling is necessary. Magnetic fields can link the crust to the core in a time scale comparable to the Alfven travel time through the core: $t_{CI} \simeq t_A = R/v_A$, where v_A is the Alfven speed.

The relation between the Alfven speed in the interior of neutron stars and the average magnetic field depends on whether neutrons and protons are normal or superconducting. In principle, the quark liquid core could also be superconducting. Therefore, we consider four possibilities when estimating the magnetic coupling time: (1) the entire liquid core is normal (i.e., the neutron, protons and/or quark liquids are non-superconducting); (2) both the neutrons and protons (or all the quarks) are superfluid; (3) the neutrons are superfluid, but the protons in at least part of the core are normal; and (4) vice-versa.

In the first case, where the core liquid is everywhere normal, the coupling time is $t_{CI} \simeq \sqrt{4\pi\rho}R/\bar{B}$, where $\bar{B}$ is the average flux density in the core. The condition that $t_{CI} \lesssim 10$ s implies that the magnetic field satisfies

$$\bar{B} \gtrsim 10^{13} \left(\frac{\rho}{10^{15} \text{ g cm}^{-3}} \right)^{1/2} \left(\frac{R}{10^6 \text{ cm}} \right) \left(\frac{t_{CI}}{10 \text{ s}} \right)^{-1} \text{Gauss .} \tag{16}$$

In this case, the magnetic field threading the core would have to be stronger than the surface field, $B_{\text{surface}} \simeq 3.5 \times 10^{12}$ Gauss (Manchester and Taylor 1977).

For the second case where all the components of the core are superfluid, the magnetic flux is squeezed into flux tubes of critical field, $H_c \simeq 10^{15}$ Gauss. In addition, the neutrons and protons are coupled by Fermi liquid effects (Alpar, Langer & Sauls 1984, Alpar & Sauls 1988). The increase in the flux density raises the Alfven speed by $(H_c/\bar{B})^{1/2}$. The constraint on the average field in the core then becomes

$$\bar{B} \gtrsim 10^{11} \left(\frac{\rho}{10^{15} \ \text{g cm}^{-3}} \right) \left(\frac{R}{10^6 \ \text{cm}} \right)^2 \left(\frac{t_{CI}}{10 \ \text{s}} \right)^{-2} \ \text{Gauss} \tag{17}$$

which is not larger that the surface field.

In the third case, the normal protons would quickly come into rotational equilibrium with the crust whereas the superfluid neutrons would take more than 10 minutes to couple (Sauls, Stein, & Serene 1982, Feibelman 1971). In the opposite case (4), the superfluid protons couple quickly via the magnetic field while the normal neutrons couple through their interactions with electrons. Assuming that the coupling between protons and neutrons is rapid, then eq. (17) also holds in this case.

4.   Conclusions

We have argued that fluid dynamic processes such as Ekman pumping do not provide the adequate core-crust coupling that is implied by the Christmas glitch. The needed coupling can be provided by a magnetic field that threads both the core and crust of the star. If both the neutrons and protons in the core are normal, the required average radial flux density in the core is $\gtrsim 10^{13}$ Gauss, about three times greater than the surface field deduced from the spin down rate. Such a strong internal magnetic field argues against the core containing only normal neutrons and protons. However, if the core did possess fields of this strength, the surface field might actually *grow* as the interior field diffuses out through the crust.

It is likely that both the neutrons and protons are superfluid and the threading field need only have a mean flux density of $\gtrsim 10^{11}$ Gauss. This constraint allows some of the surface magnetic flux to be confined to the outer crust of the star from which it could decay in $10^6 - 10^7$ years. However, since the magnetic diffusion time for fields that thread the core is $\gtrsim 10^{10}$ years (Urpin & Van Riper 1993), the decay of the crustal magnetic field would still leave a field of $\gtrsim 10^{11}$ Gauss at the surface.

It has been suggested that the magnetic flux tubes in the superfluid core may be swept out by the expanding neutron vortex lattice as the rotation of the star decreases (Ding, Cheng & Chau 1992, Srinivasan $et~al.$ 1990). Our constraints on the present internal magnetic field of the Vela pulsar indicate that the flux tubes and vortex lines manage to pass through each other. Since the present internal magnetic field is $\gtrsim 10^{11}$ G, the initial field would have to be $B_{\mathrm{initial}} \gtrsim 9 \times 10^{12} (P_{\mathrm{initial}}/1~\mathrm{ms})^{-1}$ Gauss for the sweeping interpretation to be correct. If the initial spin period of the Vela pulsar was ~ 1 ms, the flux sweeping hypothesis would require that the initial internal magnetic field be considerably stronger than the present surface field.

Future observations may provide deeper insights into the nature of neutron star interiors. Monitoring of the Vela pulsar at higher time resolution could set more stringent constraints on the internal magnetic field. In addition, XTE observations of accreting neutron stars can complement glitch observations in the study of neutron star interiors.

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Fig. 1.— Four model fits that delineate the range of acceptable parameters: dashed lines for crust-initiated models and solid lines for core-initiated models. Data for the Vela Christmas glitch are shown.